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MATHEMATICAL THINKING DEVELOPMENT STAGES OF A GIFTED 5TH GRADE STUDENT ABOUT PYTHAGOREAN THEOREM

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Abstract
The Pythagorean Theorem is one of the 8th grade achievements in the MEB (2018) curriculum. The theorem is a conceptual structure in which students have difficulty in understanding and practicing. In this context, it is aimed to get information about the mathematical thinking stages of a gifted 5th grade student. The mathematical thinking development stages of the student were examined in terms of KISS (conceptual operational symbolic process) theory. The research is case study. Yigit, who was diagnosed in the field of general mental ability and trained in science art center, was identified as the participant of the study. The name Yigit was used as an alias. Yigit is studying in the 5th grade Turkey's state school located in a province in the western region. A clinical interview was conducted with the student. This interview lasted 30 minutes. A verbal case for the Pythagorean Theorem was presented to the student. It was expected to he could developed a process of thinking towards obtaining the theorem from this verbal case. As a result of the interview, Yigit was able to draw the desired shape by following the instructions in the verbal case given. He understood that the Pythagorean Theorem was a conceptual structure obtained by the characteristics of the right triangle. He was able to develop a thinking process for symbolically showing theorem. Generalization was able to do. According to KISS theory, the process and the concept could reach the dimension of thinking together.
Keywords: Gifted student, Pythagorean theorem, Mathematical thinking.

INTRODUCTION
Gifted students can be described as individuals with special academic skills, leadership qualities and creative thinking (Davis and Rimm, 2004; Gardner, 1993; Guilford, 1967; Kirk and Gallagher, 1989; Renzulli, 2003; Sternberg, 2003). Gardner (1993) sheds light on giftedness with different areas of intelligence defined in multiple intelligence theory. However, not all intelligent individuals need to be gifted. Intelligence, talent and creativity can be considered as a whole (Karabey and Yürümezoğlu, 2015). Giftedness in mathematics can be defined as individual talent that manifests itself in mathematics (Krutetskii, 1976). Krutetskii (1976) describes these capabilities as the acquisition, processing and retention of mathematical information. Individuals with mathematically gifted abilities demonstrate their mathematical thinking skills with extraordinary speed and accuracy. In addition, these individuals can see the different relationships between the concepts (Heid, 1983). Individuals with mathematical giftedness focus on how and why a problem is solved rather than how it is solved (Sheffield, 1994). Heinze (2005) stated that gifted individuals understand quickly and make explanations more effectively than other individuals. According to Sheffield (1994), gifted individuals need to be further explored during mathematical thinking and problem solving and given opportunities in this sense in solving complex problems.

Mathematical thinking skills gain importance when examining the characteristics of individuals with mathematical giftedness. Mathematical thinking involves skills such as reasoning, modeling, prediction and proof against a problem situation (Tall, 2002). Gray and Tall (1994) explained the development of mathematical thinking with procept theory. Procept is a combination of process and concept words in English. Akarsu Yakar (2019) stated that it is appropriate to use KISS (conceptual operational symbolic process) instead of procept considering the processes contained in the word

procept in his/her research[1]. Gray and Tall (1994) explained mathematical thinking as a symbolic representation of the concept formed as a result of operation and process. In terms of KISS, mathematical thinking takes place in three stages. These stages are operation, operational process and conceptual operational symbolic process (Akarsu Yakar, 2019). If the individual realizes a concept in the learning process without realizing it, it shows the operation stage. When it starts to move to the comprehension stage in the operational process as it repeats the process, when he expresses the concept symbolically as a result of this stage, he realizes thinking in the conceptual operational symbolic process stage. The process in which the individual begins to grasp the stage as he / she repeats the process, and when he/she expresses the concept symbolically as a result of this stage, he/she realizes thinking in the conceptual operational symbolic process stage. When the studies examined in terms of KISS theory are examined (Chin and Tall, 2002; Kidron, 2008; Watson, Spyrou and Tall, 2003), it is seen that it is generally carried out on the basis of algebra, linear algebra and analysis since it includes symbolic expression process. In this research, Pythagorean Theorem is discussed and KISS theory is examined in the context of geometry. Pythagorean Theorem is one of the eighth grade achievements in MEB (2018) curriculum. Theorem is a conceptual structure that students have difficulty in understanding and applying. In this sense, this study is thought to be important in terms of demonstrating the applicability of KISS theory in the field of geometry. When the literature is examined, the researches related to mathematical giftedness are mostly related to problem solving and creative thinking (Altıntaş and Özdemir, 2012; Baltacı, 2016; Çıldır, 2017; Doğan and Çetin, 2018; Karabey, 2010; Koshy, Ernest and Casey, 2009; Leikin, Koichu and Berman, 2009; Sriraman, 2003; Van Garderen, Scheuermann and Jackson, 2013; Yazgan Sağ and Argün, 2016). In this research, the development of mathematical thinking of a gifted student is examined in terms of KISS theory. This is thought to be important for the originality of the study.

METHOD

Research Design

The research was conducted on the basis of qualitative research design. The case study was determined as the qualitative research design. The reason for choosing this pattern is to reveal the existing mathematical thinking skills of the gifted student.

Working Group

A 5th grade gifted students studying in Turkey's western region is located in a school is identified as a participant in the research. The reason why the 5th grade student was chosen by the researchers was that he had not learned the Pythagorean Theorem before and that he had the opportunity to examine the mathematical thinking processes that emerged in the process of creating a concept he did not know in detail. This student is given the nickname Yigit. Yigit's gifted diagnosis was made in the field of general mental talent and he is studying in science and art center. He has a basic knowledge of algebra and the concept of variable.

Data Collection Tools

In this study, an activity developed by Akarsu Yakar (2019) is presented to the student to form Pythagorean Theorem. It was expected to form the concept in this activity.

Data Collection

In this activity, first of all, some instructions were given to draw a figure to help him in the process of creating Pythagorean Theorem. According to these instructions, when you combine two squares perpendicular to each other with a third square, a right triangle will form. When he connects the areas of these squares and the edge lengths of the right triangle, he reaches the Pythagorean Theorem. The

[1] KISS consists of the initials of the conceptual operational symbolic process words in Turkish.

reason why Pythagorean Theorem was chosen in the research is that it is both a subject of geometry and it includes the ability of symbolic expression in the stages of KISS theory in the concept formation process. A 30-minute clinical interview was conducted with the student.

Data Analysis

The data were analyzed with descriptive analysis method. In the analysis of the data, the framework developed by Tall (2008) is considered. Thus, the process of forming the Pythagorean Theorem was examined in terms of KISS.

FINDINGS

In this research, the stages of mathematical thinking developed by Yigit, a 5th grade student with a gifted diagnosis, for Pythagorean Theorem were examined in terms of KISS theory. Yigit was able to draw the shape that he should draw in the process of creating Pythagorean Theorem correctly in his first attempt by following the instructions in the activity.

Yigit: First, I made the frames. I made those two upright. I combined the other square and the corners. I have met the last condition.

Researcher: What is the remaining part?
Yigit: Triangle.

Researcher: Can you explain how a triangle is?
Yigit: Right triangle.

Researcher: Why the right triangle?
Yigit: Because two squares are perpendicular to each other. This triangle is adjacent to them.

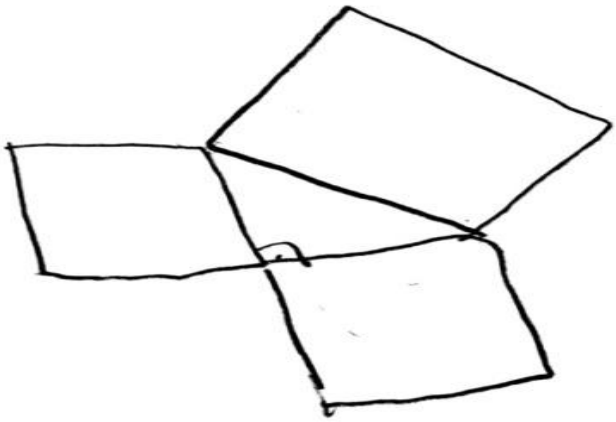

Figure 1. Figure drawn by Yigit in accordance with the given instructions

In the course of the interview, when the researcher asked what one side length of one of the squares perpendicular to one another was 3cm and the other one was 4cm, Yigit said that he could find a triangle using a ruler.

Researcher: If one of the squares perpendicular to one another had a side length of 3 cm and the other one had a side length of 4 cm, what would be the side length of the third one?
Yigit: I draw. It's 5 cm.

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(The first time he drew, he didn't say it was a right triangle.)

Researcher: How do I know that there is a right triangle?
Yigit: A parallel angle of 90 degrees. No, no parallel. Perpendicular.
(He showed up straight.)

Researcher: How do you find the areas of the quadratic regions?
Yigit: 3 cm square area 9, 4 cm square area 16, 5 cm square area 25.

Researcher: Is there a relationship between them?
Yigit: They are all square root.

Researcher: What is square root?
Yigit: None. It's a square. The square of 5 cm 25. The square of 4 cm 16.
Researcher: Is there a relationship between the areas of the quadratic regions?
Yigit: The sum of 9 and 16 is 25.

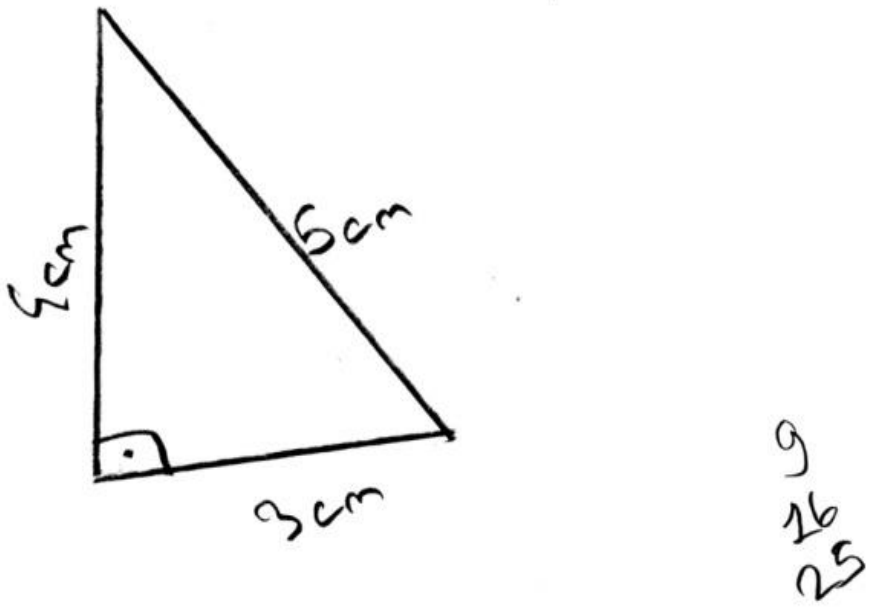

Figure 2. Solution process for Yiğit's process

The process of the interview so far shows the operation stage according to KISS theory. During the operation, the individual needs to follow only the given instructions without knowing what he or she is doing in the process of creating the concept. Yigit also showed this process. In addition, Yiğit made some mistakes in terms of language use and corrected these mistakes. Different edge lengths were asked in the following process and it was expected to realize the thinking process in the operational process stage.

Researcher: If one of the squares perpendicular to one another had an edge length of 6 cm and the other one had an edge length of 8 cm, what would be the edge length of the third one?
Yigit: I would find your areas. 64 and 36 squares. The sum of the two is 100. Hmm, 10 cm.

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Researcher: Why 10 cm?
Yigit: Because 100 is 10 squares.

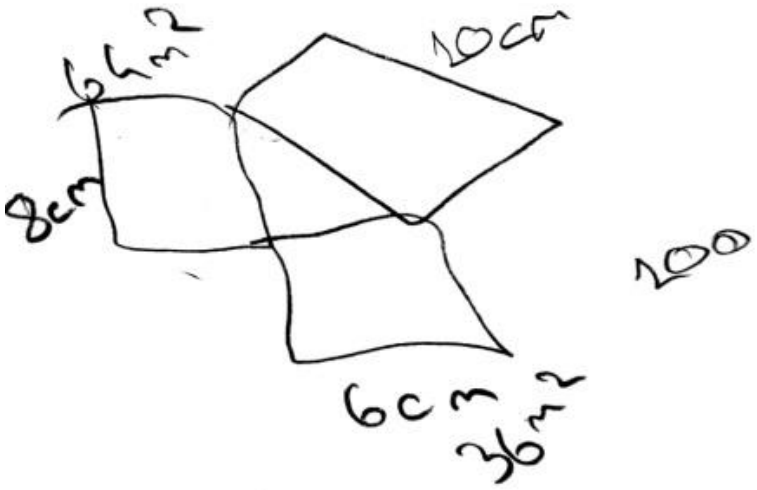


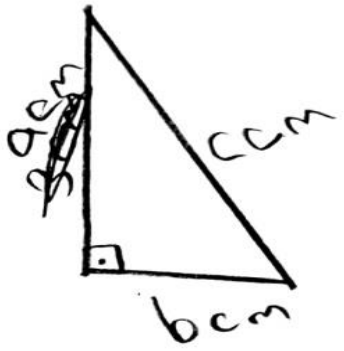
Figure 3. Solution process of Yigit for operational process

As it can be seen, Yiğit, instead of drawing the figure, applied the process he realized during the operational process stage and expressed the result correctly. After the operational process phase, the interview for the process of creating the expected concept is as follows:

Researcher: So, if you think of this solution method in terms of the side lengths of the right triangle, how do you connect?
Yigit: Area of 3. Area of 4. Area of 5. So the edges have squares.

Researcher: Can you express that?
Yigit: In the right triangle, the sum of the squares of the perpendicular edge lengths was equal to the square of the other side length.

Researcher: Draw a right triangle. If one of the perpendicular edges is a cm, the other is b cm and the third side is c cm, how do you symbolically express what you say?



$a^2+b^2=c^2$

Figure 4. Symbolic expression of Yigit for Pythagorean Theorem

As a result of the interview process, Yigit was able to express Pythagorean Theorem verbally and symbolically. He was able to demonstrate the conceptual operational symbolic process in terms of KISS.

DISCUSSION, CONCLUSION AND SUGGESTIONS

In this research, it is aimed to examine the development stages of mathematical thinking developed by a gifted 5th grade student in the process of concept formation for Pythagorean Theory in terms of KISS theory. KISS theory includes the operation, the operational process, and the ability to symbolically express the concept formed as a result of the process. Therefore, the student is presented with an activity that is expected to show these stages. As a result of the research, it was seen that the student can show all stages. Yigit was able to draw the desired shape by following the instructions in the given verbal situation. He was able to express the figure accurately and quickly. This is expected. Because gifted individuals show their mathematical thinking skills at extraordinary speed and accuracy (Heid, 1983).

Yiğit showed the operation which is the first stage of KISS theory by fulfilling the requirements of him and arithmetically revealing the processing skill. Then he repeated the skill he showed in the first stage with different number values and this time he began to give meaning to his operation. Yiğit, who fulfilled the requirements without re-drawing and reasoned in finding results, was able to show the operational process. He was able to express the conceptual structure formed as a result of the operational process verbally and symbolically. Thus, he reached the final stage of KISS theory, the conceptual operational symbolic process stage. He understood that Pythagorean Theorem is a conceptual structure obtained by the properties of the right triangle. He was able to develop a process of thinking to symbolically represent the theorem. He was able to generalize. In future researches, the developmental stages of mathematical thinking in terms of KISS theory of gifted students at different grade levels can be examined.

Yigit made some mistakes in explaining his statements. Although one of these errors verbally stated that he had drawn a right triangle, he did not visually show that the shape he had drawn was a right triangle. He showed that there is a right triangle under the direction of the researcher. In addition, while explaining that the triangle is a right triangle, he stated that a parallel angle is 90 degrees. He immediately realized his mistake; however, this suggests that some preliminary learning may have made him mistake. Because, while taking the square of the length of the value of square roots, he said. Yiğit is also studying at the science and arts center. Here, a different program is applied outside the school curriculum. He anticipates some issues. This preliminary learning is thought to cause him to confuse concepts. Therefore, it is thought that it is important to ask students to explain their statements in the lessons in order to prevent possible mistakes.

KISS theory has generally been studied on algebra, linear algebra and analysis (Chin and Tall, 2002; Kidron, 2008; Watson, Spyrou and Tall, 2003). In this research, it has been shown that it can work on geometry. Geometry includes in itself symbolization processes. In other researches in this sense, KISS theory can be examined on different topics of geometry.

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