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SELF-REGULATION BEHAVIOURS OF A GIFTED STUDENT IN MATHEMATICAL ABSTRACTION PROCESS

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Abstract

The purpose of this study is to investigate a mathematically gifted student's self-regulation behaviours while constructing and consolidating mathematical knowledge. However, the objective is to determine which self-regulation strategies influence this student's mathematical abstraction process. The case study method was used in the research. As part of the case study, interviews with a mathematically gifted student were conducted. During the interviews, the students' mathematical knowledge construction and consolidation processes were investigated through the mathematical problem-solving tasks. The coding strategy was used to ascertain the students' strategies for self-regulation while constructing and consolidating structure. On the basis of the collected data, conclusions were drawn regarding the interaction of cognitive and metacognitive components that are managed via self-regulation strategies and epistemic actions involved in the construction and consolidation of mathematical structure. It was discovered that the student was proficient at recognizing and applying prior mathematical structure through the use of cognitive strategies. The student's task-related objectives, as well as their metacognitive monitoring of the process through review of the mathematical strategies he employed to accomplish these objectives, were found to contribute to the construction and consolidation of the correct mathematical structures. The data collected revealed that the gifted student's cognitive and metacognitive self-regulation strategies are critical for the realization of the mathematical abstraction process.

Keywords: Mathematical giftedness, mathematical abstraction, self-regulation strategies.

INTRODUCTION

In mathematics, there is no universally accepted definition of giftedness. One reason for this situation is a lack of appropriate focused mathematical resources necessary for identifying students who are mathematically gifted, as well as the community's heterogeneity (Pitta-Pantazi, Christou, Kontoyianni & Kattou, 2011). While this is true, it is assumed that the characteristics incorporated into general models and classifications and believed to be associated with giftedness have an effect on their likelihood of being gifted in mathematics. This is not to say, however, that components of general giftedness adequately address mathematical giftedness (Singer, Sheffield, Freiman & Brandl 2016). As a result, the mathematics-specific components necessary for an individual to be classified can be defined as gifted in mathematics.

According to studies examining the characteristics of gifted students in mathematics engaged in mathematical activities, these individuals excel in terms of learning speed, observation and reasoning abilities (Grenees, 1981), ability to acquire mathematical knowledge, ability to process and remember this information (Krutetskii, 1976). From the perspective of mathematical problem solving, it is seen that the effectiveness, flexibility, creativity, commitment (Leikin, Koichu, Berman, 2009) and looking backstage, planning, being aware of all steps of the solution (Öztelli Ünal, 2019, Koç Koca & Gürbüz,

2021) exhibited by these students in problem solving are emphasized. Additionally, it is evident that these students' mathematical abstraction abilities are one of their highlighted characteristics. According to Greenes (1981), gifted students have strong abstraction skills, are able to make intuitive leaps, and are willing to take chances in the pursuit of new ideas. Rosenbloom (1960) states that the capacity for mathematical abstraction and generalization is one of the characteristics that differentiate students with mathematical ability from other students. However, it can be seen that gifted students' abstraction abilities in mathematics are generally considered in terms of generalization skills (Krutetskii 1976; Sriraman, 2003). Although the ability to abstract is one of the most emphasized characteristics of gifted students in mathematics, there are few studies that examine the process from a theoretical perspective. Students' and teaching methods are overlooked in gifted mathematics research studies, as was discussed by Leikin (2011), according to which teaching methods do not modern theories in mathematics education are applied. From a theoretical perspective, examining the process by which gifted students' construction and abstraction of mathematical structure can help reveal the characteristics that contribute to their effectiveness.

Abstraction in Context (AiC) is a theoretical framework for investigating students' mathematical structure construction processes (Hershkowitz, Schwarz & Dreyfus 2001). Mathematical abstraction, according to this theory, is the process of constructing a novel structure by vertically reorganizing previously constructed mathematical structures. The theory, which is based on sociocultural and epistemological principles, investigates the process of mathematical abstraction in the context of observable epistemic actions and how these actions are nested. (Hershkowitz et al., 2001). According to the theory, which proposes a methodological model (RBC + C model) for analyzing students' processes of constructing and abstracting mathematical structure, the processes of emergence and consolidation of structures are viewed as central components of abstraction. The process for constructing a new structure is defined and analyzed in terms of the epistemic actions of recognizing, building-with, and constructing. Recognizing is a term that refers to a student's awareness of a prior knowledge structure related to the current situation. The Building-with action is explained by combining and utilizing recognized structures to accomplish specified goals (Dreyfus, Hershkowitz & Schwarz, 2015). “The processes of reorganizing and restructuring what is recognized and known in order to create new meanings” are referred to as constructing (Bikner-Ahsbahs, 2004 p.120). Consolidation refers to the process by which a student becomes aware of a new structure and its application becomes direct and distinct. The consolidation process is observed by examining some of the students' cognitive and psychological characteristics. Dreyfus and Tsamir (2004) discovered that the consolidation of abstraction involves five cognitive and psychological characteristics. These are immediacy, self-evidence, confidence, flexibility and awareness. The term "immediacy" refers to both the availability of constructed structures and their direct access. Self-evidence is a term that refers to a student's acceptance of a structure without further explanation or proof. Confidence expresses belief in the student's response. Flexibility is defined as a feature of the student's mind that is based on a network of meaningful connections and manifested in the ease with which this network changes. Finally, awareness refers to the state of mind and goal-directedness of the student during mathematical activity.

The use of examples by students in formulating their thoughts and developing their mathematical language should be included in the combination when evaluating the validity of their consolidation process. Monaghan and Ozmantar (2006) assert that students require concrete examples to organize their thoughts prior to consolidation but use examples to substantiate their claims following consolidation. Additionally, additional research is necessary to fine-tune the theory's consolidation principle and its capacity for determining whether knowledge can be considered as consolidated (Tsamir & Dreyfus, 2005).

The AiC theory emphasizes the knowledge-based aspects of the individual's formation of an abstract mathematical structure, but also considers the context in which the individual exists and can influence the process (Hershkowitz et al., 2001). According to this theory, abstraction is often motivated by a need. Self-regulation, emotional gestures, and self-monitoring activities are seen as tools to be considered among the markers relevant to these needs, which are the foundation of mathematical understanding (Dreyfus & Tsamir, 2004). As a result, one could argue that the AiC theory's emphasis on these sources indicates the self-regulating nature of the construction and abstraction processes.

Panadero (2017) defines self-regulation as an umbrella term that encompasses cognitive, metacognitive, and motivational aspects of learning and takes a holistic approach to the variables affecting learning. A theoretical approach known as self-regulated learning treats the learning cycle as well as well as the various components of the cycle and looks at it from various perspectives (Azevedo, Guthrie & Seibert, 2004; Panadero, 2017). While there are a variety of models derived from a variety of theoretical perspectives, the majority of models suggest that students' use of various cognitive and metacognitive strategies to control and regulate their own learning is a critical aspect of self-regulated learning (Pintrich, 1999). Various research has validated this viewpoint, demonstrating the importance of self-regulation strategies in students' learning processes and academic performance (Pintrich & De Groot, 1990; Zimmerman, 1990). However, it is clear from the majority of these studies that self-regulation is examined as a fixed characteristic of the individual, and thus discussed in terms of aptitude. However, it has been discovered in recent years that self-regulation can be quantified in terms of events, which are defined as the examination of a moving activity by ceasing it (Winne & Perry, 2000; Boekaerts & Corno, 2005, Veenman, 2011). Due to the scarcity of studies examining self-regulation specifically, as well as studies combining new and established measurement methods, it is asserted that students will improve their perspective on evaluating the use of self-regulation strategies (Roth, Ogrin & Schmitz, 2016; Panadero, Klug & Järvelä, 2016).

According to Pintrich (1999), there are three general strategies for self-regulated learning. These are cognitive learning strategies, self-regulatory strategies to control cognition, and resource management strategies. Cognitive learning strategies such as rehearsal, elaboration, and organization can be applied to both simple recognition memory and more complex tasks requiring information comparison (Pintirch & De Groot, 1990; Pintrich, 1999). Metacognitive control and regulation strategies are self-regulation strategies for the cognitive system. These strategies include establishing a student's study objectives, developing cognitive strategies, activating prior knowledge, monitoring one's own process, and organizing one's study behaviour (Pintrich, 1999). Finally, resource management strategies refer to a student's time and research environment management (Pintrich, Smith, Garcia, & Mckeachie, 1993).

In studies of self-regulation strategies in mathematics education, problem solving plays a significant role. It is stressed that students' use of self-regulation strategies improves problem-solving performance. Schoenfeld (2007) emphasizes that what makes a student good at problem solving is not just his or her expertise, but also the strategies and self-regulation, tracking, and control elements that the student employs to apply that knowledge. One of the points emphasized during the problem-solving process is the importance of metacognitive actions as a component of self-regulated learning. Garofalo and Lester (1985) assert that focusing exclusively on cognitive analysis is insufficient to improve students' mathematical performance and that metacognitive decisions play a critical role in influencing cognitive actions.

From the standpoint of self-regulated learning, studies examining the self-regulation strategies of gifted students in mathematics draw attention to the fact that it is dealt with in the field of problem solving and that gifted students are mostly compared to other students. According to Zimmerman and Martinez-Pons (1990), gifted students demonstrate significantly greater verbal efficiency, mathematical efficiency, and strategy use than average students. Swanson (1992) discovered that talented students solve problems with fewer moves and have a higher level of metacognitive knowledge than other students. Montague and Applegate (1993) discovered that gifted students verbalized more than students with average performance and learning disabilities in problem solving processes by contrasting cognitive and metacognitive verbalizations. These studies undoubtedly contribute to our understanding of the characteristics that distinguish gifted students from others. On the other hand, studies examining gifted students' self-regulation within the context of their unique giftedness and self-regulation dynamics are necessary (Pintrich, 1999; Efklides, 2019).

The aim of this research is to investigate the constructing and consolidating processes of a mathematically gifted student. The epistemic actions of the student in case of problem solving were examined through the RBC+C model. Additionally, it is intended to ascertain the self-regulation strategies that influence this student's abstraction processes. The effect of the student's strategic behaviors in problem solving on the emergence of epistemic actions is discussed. It may be beneficial to consider the interaction between knowledge-based actions and the use of self-regulation strategies when observing the epistemic actions presented by the RBC + C model and detailing the contextual structures that may affect abstraction. Examining gifted students' mathematical construction and consolidation processes within the context of self-regulated learning and AiC theories may also contribute to the field of gifted students in mathematics by elucidating the characteristics that enable these students to succeed in these processes.

METHOD

This research is a portion of a qualitative study conducted with gifted mathematics students. The purpose of this study is to investigate the processes by which students construct and consolidate mathematical structure, as well as the self-regulation strategies that students employ during these processes. According to Yin (2003), the case study is used to investigate a phenomenon in real-world settings with a variety of data sources. As a result, the case study method was chosen for the study.

Typically, data for a case study are gathered through observation and interviews (Büyüköztürk, Çakmak, Akgün, Karadeniz & Demirel, 2009). In this context, interviews with the research participant were conducted. In the interviews, open-ended mathematical problems were used to collect data. Additionally, during the problem-solving sessions, the student's structure constructing-consolidation processes and self-regulation strategies were observed.

Participant

The researcher, Nil (pseudonym), continues to a Science and Art Center in Turkey, where gifted students receive education outside of school. The study's "Criteria Scale for Determining Potential Giftedness in Mathematics" (CSPGM) was administered to forty eighth-grade students at Science and Art Center. On this scale, Nil performed admirably. Nil's selection as a participant was also influenced by her ability to speak and her willingness to volunteer for the study. As a result, the participant was identified using a purposive sampling strategy.

Data Collection Instruments and Procedures

To select participant for the study, a criterion scale (CSPGM) was developed to assess potential mathematical giftedness. The scale is comprised of twelve open-ended mathematics problems. The criteria set out in Krutetskii (1976) and Niederer, Irwin, Irwin and Reilly (2003) studies, which will allow to classify gifted students in mathematics in problem solving situations, were taken into consideration in the creation of the problems. As a result, students must:

1. Recognize the patterns and rules that will lead to a solution.
2. Understanding and reversing processes and procedures.
3. Converting the problem's situation to mathematical representations.
4. On the basis of the information provided
    a) Spatial reasoning,
    b) Probabilistic reasoning,
    c) Logical reasoning and
    d) Demonstrate permutational reasoning

Problems have been created in order to determine their properties. The problems' validity and reliability were established through expert consultation and a pilot study.

A scoring key was developed for each question based on the review of the students' answers to the problems. As a result, the accuracy value assigned to each question ranges between 0 and 4. The scale yields a score between 0 and 48. Students with scores of 0-15 were classified as low-performing, 16-31 as medium-performing, and 32-48 as high-performing, based on their responses to the scale's problems. The responses of the students were analyzed by the study's first author and a mathematics instructor. Inter-rater agreement was found to be 0.92.

The research's case participant is Nil, who earned 44 points in CSPGM. The case study used open-ended problems to observe students' structure constructing - consolidation styles and self-regulation strategies during these processes. Expert opinion and a pilot study were used to ensure the case studies' validity and reliability. The first author applied the case study problems in two separate sessions and recorded Nil's problem-solving processes. The RBC + C model was used to analyze the processes of construction and consolidation.

The aim of the first session's problems was to observe Nil's processes of recognizing, building with and constructing. Nil completed two problem-solving tasks during the first session. (See Figures 1 and 2) Here are the Task A and Task B problems.

A1. Find the smallest positive integers that do not completely divide the following numbers.
i) 10 ii) 36 iii) 120 iv) 210

A2. Which of the following numbers is the smallest that does not divide a positive integer exactly? Explain.
i) 6 ii) 7 iii) 9 iv) 10

A3. Assume that the "smallest non-divisor" of a number is the smallest positive integer that does not divide an integer exactly. What properties does a number need to have in order to be the "smallest non-divisor"?

Figure 1. Problems used in Task A

B1. Which natural number between 1 and 30 can be the "smallest non-divisor" of any integer?

B2. Determine the numbers that, even if they are not prime numbers, can be the "smallest non-divisor" of a number between the natural numbers 1 and 100.)

B3. Find the two smallest natural numbers whose "smallest non-divisor" is 5?

B4. How many natural numbers have two digits and the smallest non-divisor is four?

Figure 2. Problems used in Task B

According to Dreyfus et al. (2015), consolidation is concerned with examining student actions following the emergence of a construct, and in order to investigate consolidation, the analysis typically proceeds forward from the end of the constructing action. As a result, Nil was assigned the following Task C during the second problem-solving session held one week after the first session in order to observe the processes of consolidating the structures she constructed during the previous session (See Figure 3).

The teacher has some pens. He intends to distribute these pens evenly and completely among his students.
The teacher performs sequential and independent calculations to determine which shares will be successful and which will be unsuccessful among the various numbers of students in the allocation plan he designed.
For instance, if the teacher has 24 pens, he or she can share them equally with two students, three students, or four students without having a pen left over; however, if the teacher wishes to share with five students, he or she cannot share in equal numbers.

C1. If the teacher has at least how many pens, for the first time, the allocation plan fails to distribute them evenly among seven students? Give an explanation for your answer.

C2. Is it possible that the Teacher's plan will fail to share between 6 students for the first time for any item and number of students? Give an explanation for your answer.

C3. Assume the number of items is unknown and there are fewer than 50 students. Determine the possible number of students who will fail to adhere to the teacher's plan. Give an explanation for your answer.

Figure 3. Problems used in Task C.

The self-regulation strategies used by Nil during the construction and consolidation processes were determined during the problem-solving sessions. The strategies were determined using Marcou's (2007) coding scheme. Self-regulation strategies incorporated into the coding scheme; It is classified as cognitive, metacognitive, and resource management. The rehearsal, elaboration, and organizational strategies are all types of cognitive learning strategies. Metacognitive strategies include self-monitoring and self-regulation. Since this study involved individual interviews with students, resource management strategies were not coded. The coding scheme was piloted using case study problems. The scheme was used to code and analyze the student's discourse while construction and consolidation processes. To ensure the analysis's consistency, student discourses were coded twice, at distinct times. Incompatible coding is omitted from the analysis.

RESULTS

The following are excerpts from Nil's interview response to Task A's first question (A1).

(**R:** Researcher, **N:** Nil)

> **4N:** *Hmm (considers)... I need to find out how many divisors these numbers have... wait a minute, I don't need it. It states that the smallest integer that does not divide exactly (Circles the word "smallest") ... Then three (for ten) ... 2 for 15 I set it aside for a moment... 2, 3... (trying out different numbers) 5 (for 36) ... 120 for... 1,2,3,4,5,6 (trying the numbers) 7 is ok...*
>
> **5R:** *I suppose you try for any possible number.*
>
> **6N:** *Yes, for 210, 1, 2, 3... (quickly trying the numbers) 4, well.*

Nil obtained the correct answers to the first question (A1) through testing. He stated in subsequent sections of the interview that he conducted these experiments in accordance with divisibility rules. Consider the section where it tests that the smallest positive integers that do not divide a positive integer are or are not 6 and 7 (Problem A2).

> **15N:** *All right... (reads aloud the problem A2) ... Hmm... As in the preceding question?*
>
> **16R:** *Is the same question being asked?*
>
> **17N:** *Hmm... that means it will be divided by all of 1,2,3,4,5 (thinks for 6)*
>
> **18R:** *So, what are you going to do about it?*

19N: *... must test each one individually. Alternatively, in accordance with the divisibility rules... (contemplating). Could it be sixty? (She inquires) 60 is divided into six, though that is not the case... It would not be 40 or something like that; there is 80 but it is not divided into three equal parts; 90?... However, when divided by two and three, isn't it divided into six? (Inquires of herself) ... how is it going to be? If that is not the case, then 6 could not be possible (Confident)... However, I believe that 7 could be the case. Because, as we are aware, there is no such rule in this, as for 6. I don't have a specific number in mind at the moment, but I believe it might be (7). (taking notes aside).*

20R: *What if we wanted to locate a particular number?*

21N: *... For instance, 60 is divided by 2, 3, 4, 5, and 6. As a result, it becomes 60.*

Nil believes that in order to determine whether the number 6 is the "smallest non-divisor," she should conduct tests using the divisibility rules for the numbers she has determined, expressing the preliminary information that the numbers that can be divided by 2 and 3 will also be divided by 6 (19N), which she recognized. Additionally, Nil predicted the number 7 based on the knowledge structure she possessed. When he was asked for a sample to support this prediction, he provided a valid response by making a selection to be divided by positive integers prior to 7 (21N). This situation may indicate that Nil is developing an opinion about finding the smallest non-divisor for a desired number (for example, the smallest non-divisor is 7 for 60). This section of the problem will discuss how the knowledge she possesses will manifest itself in the ongoing process.

It is clear that Nil did not conduct tests as she did in previous chapters, but instead provided an answer in her own words based on the divisibility rules to the question of whether the number 9 can be the smallest non-divisor of any number. This means that Nil began to devise a strategy based on the structure she had. Although he appears to be certain of his answer, he attempted to find a number whose smallest non-divisor could be 9 in order to support it. While building with, the student can verify the accuracy of an idea or hypothesis proposed for resolving the problem. As a consequence, the current state of affairs means that Nil is currently building with it. Let's take a look at the relevant part of the interview.

22R: *Could it possibly be 9 (the smallest non-divisor of any number)?*

23N: *(quickly responds) Now, anything divided by nine must be divided by three, but there is no rule that every division by three must also be divided by nine, so it could be nine (Confident). Shall I consider numbers?*

24R: *Do as you please. If you wish to contribute...*

25N: *So, for instance, there is 420 from 60 times 7, or 30 times 7 because 30 cannot be divided by 7. Then it's 210, but the seven is (multiple), so... 210 cannot be divided by eight. We have already stated that the range is from 4 to 210. Then, no... If I consider 420... That number would be 9 420. That is because 420... 2,3,4,5,6,7... but not by eight. Then... (Contemplating)... As if I need to try something else... Hmm... Let me think of a hundred... It is not divided into eight sections. If I say 1000, it is divided into eight pieces, but not by seven... When I say 7000, it is not divided into three numbers. It will not be divided into 7 if I say 3000. Let's say I say 21000. It is also divided by 2,3,4,5. There are two and three separated by six, divided by seven, and divided by eight. Don't be separated by 9 if anything happens. Is it split? The sum of their numbers is not split. Really, it's 21000. Oh, that's great. I've discovered it.*

When the section in which Nil justifies her answer that 9 may be the smallest non-divisor of any number is examined(25N), it becomes clear that she attempts to form a number by experimenting with all positive numbers preceding 9 as factors. Considering that Nil recognized her knowledge of divisors of a number and which she used this knowledge by testing, both epistemic actions (recognizing and building-with) occur concurrently. This situation demonstrates the nonlinear and nested nature of epistemic actions. Nil is in the process of building-with it in the section where 10 states that it cannot be the smallest non-divisor of a number.

> **26R:** *So, do you believe it could be 10 (the smallest non-divisor)?*
>
> **27N:** *21000 multiplied by 9 but divided by ten.... (Contemplating)*
>
> **28R:** *What are your thoughts?*
>
> **29N:** *... Now, if the end is 0, it will also be divided by 5.... If not, then in this case, 10 cannot be the smallest non-divisor (confident).*
>
> **30R:** *Do you believe the ones with the smallest non-divisions share any characteristics?*
>
> **31N:** *Hmmm... I'll say it now... It cannot be 6, as it is dependent on 2 and 3. I believe it should not be contingent upon another number. For instance, 7 is either directly divisible by itself or not. Not at all like 6. For instance, not ten, because any number divided by ten is divisible by five. However, 9 appears to refute this. Because every number divisible by three is divisible by nine... (considers) No, quite the contrary. If it is divisible by three, it becomes nine. Then, indeed. Correct. As I previously stated... precisely. Then I'll be able to say...*

Another point that stands out in this section of the interview is Nil's consideration of the common characteristics of numbers with and without the smallest non divisors (31N). This circumstance may indicate that it has begun the process of construction concurrently with recognition and built with. Nil's assertion that 6 and 10 cannot be the smallest non-divisors because they are the product of two prime numbers is a critical component of the new structure she will construct.

The following section of the problem (Task B) required Nil to determine which numbers between 1 and 30 can be the smallest non-divisor. Using the common feature (31N) mentioned in the previous section of the interview, Nil correctly classified the numbers by attempting all of them. Rather than determining random numbers in her tests, she advances by attempting to construct numbers that include all the numbers preceding the number she is investigating as a factor, regardless of whether the smallest non divisor exists or not. She continued her operations while doing so by writing new multipliers based on the number she investigated. Nil has performed the following operations at this point (See Figure 4).

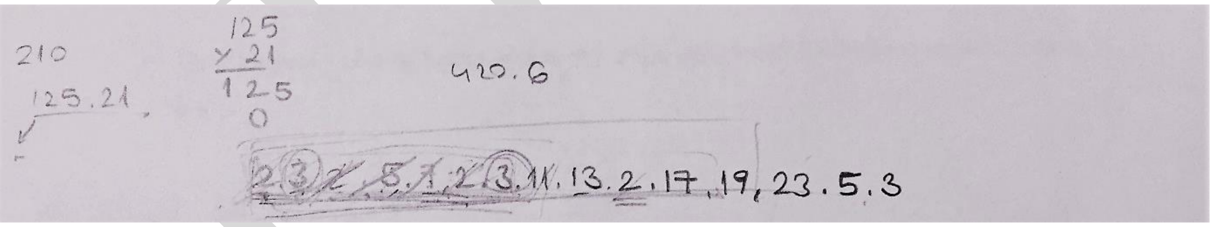

Figure 4. Nil's operations.

After some time, Nil began concentrating on the common characteristics of these numbers when classifying them with and without the smallest non-divisor. In this case, it is possible that Nil will need to construct a new structure. Consider the dialogue in the pertinent section.

> **39N:** *Not 24. True, this will not be the <u>first non-divisor</u>. Indeed, these appear to be prime numbers, but there are eight and nine. Did I write it incorrectly? (Taking a look at the numbers)*
>
> **40R:** *Do you believe it was incorrect?*
>
> **41N:** *Indeed, they are right. Could there be numbers that are both squares (a perfect square refers to natural numbers) and primes? (She inquires) ... I believe I will continue until thirty... (She is resuming her trials) ... Let us repeat this procedure for 27. (She investigates the*

multipliers depicted in Figure 4) Again, there are three; we require three more. Then it could be 27. Then it's not just square numbers...

42R: *Has your theory been refuted in this instance?*

43N: *Correct... because there are others besides the square.*

44R: *Are you going to continue?*

45N: *I believe I'll finish by 30. I'll check later. There are four and seven for 28, so it cannot be twenty-eight. It changes to 29. As I previously stated, there are prime. It seems as if my theory is corrupt, but here (showing the smallest non-divisors) there is no prime, let's see… it will be 29. It's not 30 for 30, it's already 2, 3 and 5, so it's not 30.*

46R: *Well, will you look at their common features again?*

47N: *Yeah, let me take a look at it now.*

48R: *Okay.*

49N: *For instance, there are: two to the power of one... two to the power of two , two to the power of three, two to the power of four. two to the power of five, 32 is not included on this list, but it most likely would be. And… three to the power of one, three to the power of two, three to the power of three. For 4... four to the power of one, four to the power of two. five to the power of one, five to the power of two… The squared and cube of 7, 8 and 9 will not be in this range (1 and 30 range). So, how about we put it this way? Are they prime numbers and any number's squares and cubes?*

50R: *Have you checked it against your numbers?*

51N: *Exactly; I believe squares and cubes of prime numbers, but there is 16... or 27...*

52R: *What are your thoughts?*

53N: *A prime number or a prime number's powers, such as 2,3,4,5. Or, to put it another way, it must be a prime or a power of a prime. There must be only one prime number; otherwise, the smallest non-divisible number, as in 6, cannot exist.*

54R: *Do you believe it fits everyone?*

55N: *Yes, (Confident).*

Another point that stands out in this section of the interview is Nil's work on developing a language he refers to as "the first non-divisor." After establishing the cases in which a number cannot be the least divisor in the preceding chapters, the following sections will discuss the language's use in the ongoing process. As demonstrated above, it has resulted in the construction of new structure regarding the properties required to be the smallest non-divisor. As a result, the Nil's structures can be expressed as follows in this section.

- If a positive integer can be written as the positive powers of any prime number, it can be the smallest non-divisor.
- If a positive integer can be written as the product of the positive powers of two or more prime numbers, it cannot be the smallest non-divisor.

In the second problem of Task B, Nil was asked to decide the numbers between 1 and 100 that can be the smallest non-divisor of any integer, even though they are not prime numbers. One could argue that Nil makes effective use of the information she generated in the first chapter. Consider the pertinent section of the interview.

56R: *Okay, then, how about we move on to the next question?*

57N: *Let us proceed... (continuing reading question B2). Now... Uh... well, here are the prime number powers... For instance, 16 is not a prime number, but a power of two. I do not begin with the number one because it is not a prime number. The other powers of two (apart from 1) are 4,8,16, 32,64. For the powers of three, it would be 9, 27, and 27 multiplied by three equals 81.81. The ones following 81 do not. I can't think of four, that is not prime. If we consider 5, it will not be until after 25. If I consider 7, would be 49. Even if I say 11, it will pass. That is all.. (checking). That is correct (confident).*

As can be seen from her solution, (57N) Nil clearly employs the structure that a prime number or a positive power of a prime number is the smallest non-divisor in order for a number to be the smallest non-divisor, which is one of the newly constructed structure.

Nil was asked to find the two smallest positive numbers with 5 as the smallest non-divisor and all-natural numbers with two digits with 4 as the smallest non-divisor in problems B3 and B4. Despite the fact that Nil has created the structure that will allow her to answer these questions, she chose to arrive at the desired numbers by conducting tests in the beginning. This section's conversation dialogues are listed below.

58R: *Well, if you're confident in your response, let's move on to the next question.*

59N: *All right... (aloud reading of question B3) Hmm... not 6. It cannot be subdivided into four parts. When I say 8, it is not divided into three parts. Now, just a moment... I said six, or if I multiply it by four, it equals twenty-four. However, let us see if there is a smaller one... Consider the number twelve. Let us investigate (checking). Yes, that would be twelve. Let us investigate whether there is a smaller one. 11 is not possible., but also not 10. Therefore, let us investigate. So the ones following the smallest 12. So let`s look numbers after 12. It's already from 12 to 24; otherwise, it'll be 24. Not thirteen, not fourteen, not fifteen... But what if I don't try each one one at a time for a second? But let's check. It will be 24, so let's give it a shot. Not 16, 17, 18, or 19 in any case, nor 20, 21, 22, or 23. Yes, there are 24 of them. As a result, 12 and 24. Actually, I shouldn't do that because if she says 50, it would be difficult to try each one individually. That is how I believe it can be achieved (she means doing it by adding a multiplier).*

60R: *For instance, I believe there is a situation like this in the next issue.*

61N: *(Reading Exercise B4) ... Yes... (laughs)... Then we can employ that strategy. Let's find out and extend the first one. It is 6, if we consider the first one. Since it says two digits, I'll calculate it right now. We don't get 12 when we add 2 (multiplier). Since it's divided by four, but times three equals 18. In this case, I discovered the smallest, which was 18. It's lovely. We can't tell 4 times... because it's split by 4. It would if I say it five times. It just so happens to be 30... Yeah, yes, If I say 6, I don't get another two. Let's see, if I say 7, 42. I'm afraid I won't be able to tell 8. If I say 9, it will be 54, and I will be able to write 54. I wouldn't put it at ten. It will be 66 if I say 11. I'm afraid I won't be able to tell 12. If I say 13, what is 13 times 6 (calculated) 78? That's perfect. Is it split into four parts? It is indivisible. 14 is no. The number two appears... I mean, I will do it for odd multiples. I believe it would be 90 if I said 15, yes, When I say 17, I've already gone over two digits. That's what there is to it.*

62R: *Do you think it's possible that it's the number you skipped in between?*

63N: *I think it's fine because I used the same method in the previous question. It can't possibly be missing, in my opinion. (Self-assured.)*

Nil chose to reach the numbers by trial when attempting to find the smallest two natural numbers with the smallest non-divisor of 5, despite the fact that the solution can be found by writing the factors preceding 5. (59N). She is aware, however, that the method he employs will make the solution more difficult in larger numbers. The fact that Nil chose not to use the structure she constructed and instead pursued a "multiple" path to a solution, as she refers to herself, may indicate that the newly constructed structure is fragile. However, she recognized that testing would make the solution more difficult in the following question and chose to use the newly constructed structure to write the necessary factors to obtain the desired numbers (61N). Nil stated that she was certain of her answer (63 N).

What has been shared thus far demonstrates that Nil generates mathematically correct knowledge. Now, let us take a look at how Nil has demonstrated her self-regulation strategies throughout the interview.

When testing the problem-solving process in general, it becomes clear that Nil is cognizant of each step she takes to arrive at a solution, beginning with the first question. As soon as she began solving the problem (4N), Nil circled the keywords and separated the relevant and irrelevant methods from the strategies she considered to arrive at a solution (4N), She took notes as necessary (4N, 19N, 25N), demonstrating that she uses cognitive rehearsal strategy. She demonstrated that she used the cognitive elaboration strategy by expressing the problems in her own words (4N, 57N) and developing some plans and verbally expressing them (4N, 19N, 25N, 45N) during the problem-solving process's initial stage. As illustrated in Figure 1, to express his thoughts in her tests, she created a representation of the numbers that included all the numbers preceding the multiplier, regardless of whether it was the smallest non-divisor or not, and reached the conclusion by constantly adding multipliers to this representation, demonstrating a cognitive organisational strategy. It's worth noting that Nil checked her solutions throughout the problem-solving process and validated them through review. (25N, 39N, 43N, 49N, 51N, 53N,59N, 61N). This provides critical evidence that Nil employs a metacognitive regulation strategy. Similarly, she frequently cast doubt on her conclusions (19N, 57N, 59N), sought out novel solutions to bolster her case (41N, 59N,61N) as a result, she metacognitively followed her own solution process.

One week after the initial interview, the details of which were shared previously, another session was held. The purpose of this session was to ascertain how Nil used the structure she has already gathered. Nil's solution to the problems (Task C) provides insight into whether the information about the smallest non-divisor is consolidated by various scenarios involving the distribution of items in a teacher's hand among her students.

She quickly recognized the knowledge she had constructed in the previous interview after reading the Nil problem (C1).

> **67N:** *(reading problem) ... Okay, fine. So, as we previously stated, the first divisor of this number will be 7. (he notes under the problem)*
>
> **68R:** *Is that a question about that subject?*
>
> **69N:** *(rereading the question) ... True (Confident)*
>
> **70R:** *How are you going to find it?*
>
> **71N:** *We are asked to find the smallest number whose first non-divisor is 7. Allow me to repeat the procedure I used previously. (She means calculating the result by multiplying it by a factor.) Let me do it this way: 2 times 3 times 2, here I need to add 5 'to divide by 5, so 60 (multiplies).*
>
> **72R:** *Isn't it possible to have fewer pens?*
>
> **73N:** *No way, (Confident). There should be at least 60 pens available.*

Nil has made it clear after reading the question that she should use the structure she previously constructed to find a solution (67N). While doing so, it is worth noting that he employs a concept he refers to as "first non-divisor." Along with acquiring the necessary structure for resolving the problem, Nil had begun developing this problem-related language during the previous session. The fact that she immediately recognizes and expresses her prior structure after reading the problem demonstrates the immediacy of the act of consolidation. Additionally, Nil responded to the researcher's question about whether fewer pens could be included (73N) and did not feel the need to check her response, despite the fact that she could. This situation demonstrates that Nil possesses another characteristic of the consolidation action: self-evidence.

Now consider Nil's solution to the C2 problem.

> **75N:** *(reading the problem C2) ... independent of the previous question, I suppose?*
>
> **76R:** *Correct.*
>
> **77N:** *All right... (re-read the problem) ... Acceptable. Allow me to repeat the procedure (which entails writing the multiplier as in the previous question) ... However... Thus, the question is, "Can 12 be the first non-divisor?" ... Now... What did we just say? (Inquires of himself). It must be a prime number or one of its powers. 12 is not a prime number, nor is the power of prime numbers. As a result, it cannot be. When we write the factors, we see that when 3 and 4 are present, 12 is required. As a result, it cannot be 12.*

After reading the problem, Nil restates it in her own words. Then he expressed the structure that he had previously formed in a clear and explicit manner (77N). In the light of her response to this structure, it's significant that she illustrates her ideas with an example. Before consolidating, student require concrete examples to formulate their thoughts; however, they use examples to illustrate their assertions following consolidation (Monaghan & Ozmantar, 2006). This perspective reveals an important clue that Nil consolidates the structure she has created.

Finally, let us look at Nil's C3 solution. As she did in the others, Nil re-expressed the problem's statements in her developed language and made explicit use of the structure she had constructed.

> **81N:** *(reading aloud the problem C3) ... This does appear to be a bit of a challenge... As stated here, it must be the first non-divisor. This is referred to as "plan failure"... Now that we don't know how many pens there are and the number of students is less than 50 (he is rereading the question), it considers the numbers less than 50 for the number of students and wants us to find the ones that do not divide first... Ha The question actually asks us to find the numbers which can be the first non-divisor that is less than 50. I am not required to attempt each one individually.*
>
> **82R:** *How will you then find it?*
>
> **83N:** *There is no need to test of prime numbers and powers. It's a pleasant experience (Laughs). Let me begin by writing the prime numbers from 1 to 50... 2,3,5,7,11,13,17,19,23,29,31,37,41,43 and 47. Then, let me consider their powers. All powers up to 50 are now multiplied by 2, 4,8,16,32… 9 and 27, 81 passes. We can substitute 25 for 5, then add 49 to complete the equation.*
>
> **84R:** *Are you certain about the numbers?*
>
> **85N:** *I'm certain they are.*

While Nil claims that this is a strategy of testing in order to solve the problem, it appears as though she prefers to use the structure she has generated (81N). Her frequent reference to the structure she has created when utilizing her knowledge can be interpreted as an indicator of "flexibility." Furthermore, her conclusion, which uses this structure smoothly and effortlessly, can be viewed as an important sign of "awareness."

Let's return to the self-regulation strategies demonstrated by Nil in the context of the consolidation process. Nil made a direct reference to the structure she constructed after reading the problems, noting that "the first divisor will be 7" under the query (67N), demonstrating the strategy of cognitive rehearsal. She did, however, redefine the problem in her own language by coining the term first non-divisor (67N, 71N). Additionally, given that she explained her plan by stating that she should approach the question similarly to how she did in previous activities (71N, 77N), it is possible to conclude that Nil employs a cognitive elaboration strategy. Another remarkable aspect of Nil's performance in this session was her repeated reading of the problems (69N, 77N, 81N). Given that Nil, who is certain she has gathered all necessary information for solving the problem, attempts to recall her previous solutions (77N, 83N), it can be concluded that she organized herself metacognitively. Consequently, while Nil is confident in her solutions, she checked her answer use other mathematical strategies she had previously use to justify her answer (77N) and metacognitively monitored the solution process. When the process is viewed holistically, the question arises as to whether there will be an interaction between Nil's self-regulation strategies and the consolidation action's characteristics. This question will be revisited in the research's discussion section.

DISCUSSION and CONCLUSION

Nil's construction and consolidation processes were observed in this study while she was engaged in problem solving activities. Additionally, her strategic behaviours are discussed in terms of self-regulation during these processes. This situation enabled the investigation of multiple factors affecting the realization of mathematical abstractions concurrently. This section emphasizes, first and foremost, the emergence of Nil's epistemic actions and strategic behaviours during the abstraction process.

In Nil's construction and consolidation processes, cognitive learning strategies are one of the strategies that investigate self-regulation behaviours. These are strategies that entail taking into account critical information about the task at hand, disclosing ideas, and establishing a connection to existing knowledge (Pintirch & De Groot, 1990; Pintrich, 1999). According to Winne and Perry (2000), this strategy is primarily based on representations from previous tasks and information from long-term memory.

Nil, who makes extensive use of cognitive strategies, has made connections with existing knowledge structures while solving problems and has progressed by making plans to built-with this relationship to solve the problem. Considering the RBC + C model's behaviours that students present in their recognizing and building-with actions, the presence of cognitive learning strategies becomes apparent. Indeed, the research findings corroborate this assertion. Nil, for example, used a cognitive rehearsal strategy to consider her prior structure of divisibility rules and associated it with the problem situation. As a result, the use of cognitive strategy aided Nil's recognition of the previous knowledge structure. Nil shared her ideas and plans for determining whether the given numbers could be the smallest non-divisor using both a cognitive elaboration strategy and the mathematical strategies she devised. This demonstrates the critical role of cognitive strategy-related behaviours in recognizing and building-with actions.

Not only at the start of the problem-solving process were cognitive rehearsal and elaboration strategies observed. Nil appears to use these strategies in her problem-solving process with each new structure she recognizes. From this perspective, it has been concluded that Nil's cognitive strategies are heavily utilized during recognizing and building-with actions. This result does not imply that Nil did not engage in behaviour consistent with the use of cognitive strategies during the constructing. However, one could argue that behaviours associated with metacognitive strategies evolved concurrently with observations of behaviours indicative of the construction of the new structures.

Theoretical studies on self-regulated learning and research on mathematical problem-solving both emphasize the importance of cognitive regulation strategies. These strategies, referred to as metacognitive regulation and metacognitive monitoring, are viewed as the gateway to self-regulation in the learning process (Winne & Perry, 2000). These strategies make a brief reference to the student's behaviour and comprehension in the learning environment, allowing him or her to make adjustments based on a review of their own goals and criteria. Nil frequently employs these strategies in the process of construction, as has been observed. Nil's mathematical strategies were deduced, and it was discovered that she was in complete control of each step she took, examined the consistency of the solutions they discovered with previous answers, and devised and applied alternative methods for verifying these solutions. Consider the episode where the Nil determines the numbers (between 1 to 30) that are and are not "smallest non-divisor". Nil conducted experiments by reminding herself of the paths she took in previous chapters, evaluating the reasonableness of her conclusions, and attempting to generalize her findings for each issue she worked on. From this vantage point, the traces of Nil developing her new mathematical knowledge were observed in conjunction with indications of metacognitive regulation and monitoring strategies. Using Nil's solution processes as an example, it is clear that metacognitive regulation and monitoring strategies contribute significantly to the construction processes. Of course, it cannot be claimed that only the existence of these strategies is effective in constructing mathematical knowledge. Within the confines of this research, the abstraction process of a gifted student is discussed in a cognitive and metacognitive perspective. It should be noted that self-regulation addresses more than pure (meta)cognitive strategy, and that motivational and affective components of the student can influence the process (Efklides, 2019). However, when Nil's behaviours are examined, it is thought that the effect of employing these strategies cannot be ignored.

Another area where clues about Nil's use of self-regulation strategies were discovered was the process of consolidation. When investigating Nil's coded discourses, cognitive strategies provide more specific clues than metacognitive strategies. For instance, after reading the problem presented in the second session, he referred to his prior structure, interpreted the problem in her own words (first non-divisor), expressed her solution plan, and took notes, using the statement "... as we did previously, that is, the first non-divisor of this number will be 7..." These intimations suggest that Nil makes use of cognitive rehearsal and elaboration strategies. In light of the characteristics of the consolidation action identified by Dreyfus et al. (2015), it is believed that Nil's emergent behaviours as a result of constructing cognitive strategies contribute to the formation of immediacy in terms of direct access to the structures she constructs, as well as flexibility in her references to the structures she constructs in the process. Nil attempted to validate each step she took toward a solution during the first session, throughout which she constructed a new structure. She was not required to verify the information structure she confidently and smoothly used in the second session, but she was able to use alternative methods when justification was requested. The scenario outlined here is consistent with previous research on the consolidation process. Thus, Monaghan and Ozmantar (2006) emphasized that while students required concrete examples to formulate their thoughts prior to consolidation, they used examples to demonstrate claims following consolidation. Nil's monitoring behaviours during the second session, during the consolidation process, can be interpreted as her utilizing metacognitive strategies to check the consistency of her solutions, but also to justify her claims.

The cognitive and metacognitive components of self-regulation strategies, as well as the epistemic actions involved in constructing and consolidating the knowledge structure, are thought to interact. This interaction can be summarized as follows in light of the findings from the problem-solving sessions with Nil.

Cognitive self-regulation strategies require consideration of existing information about the task being studied (cognitive rehearsal), the production of answers by establishing ideas and plans for the solution (cognitive elaboration) and the combining of task-related information (cognitive organization). Therefore, the use of these strategies affects the whole process, but especially accompanies the realization of recognizing and building-with actions. On the other hand, metacognitive self-regulation strategies require a review of the acquired structure about the task in relation to the objectives (metacognitive regulation), as well as calibration of the obtained results or the entire process (metacognitive monitoring). While metacognitive strategies are visible throughout the process, their presence becomes apparent when mathematical strategies related to the solution are implemented. Indeed, Marcou (2007) observed a cyclical aspect of self-regulation, stating that as the stages of self-reflection from intuition progressed, the number of metacognitive strategies increased while the number of cognitive strategies decreased. The findings of the study corroborate this assertion.

Within the limitations of the study reported in this article, it has been determined that strategic behaviours are critical in the constructing and consolidation of the mathematical structures. This finding is consistent with previous research (Garofalo & Lester, 1985; Schoenfeld, 2007), which asserts that not only cognitive competencies are effective in mathematics performance, but also strategic behaviours and metacognitive decisions. However, Nil's findings regarding the abstraction process, which constructs an ideal profile of her strategic behaviour and mathematical ability, are strikingly similar to those of studies on gifted in mathematics from a self-regulation perspective. Nil's performance was not compared to that of another student during the problem-solving process. On the other hand, his performance in this process is consistent with the characteristics highlighted in studies comparing gifted students to other students (Zimmerman & Martinez-Pons, 1990; Montague & Applegate, 1993).

When Nil's abstraction process is investigated in the perspective of strategic behaviour, it is noted that self-regulation and problem-solving strategies interact. Indeed, her awareness of the steps she has taken for the solutions, planning and looking backstage offer important clues as to her use of problem-solving strategies. This finding is consistent with the findings of studies that deal with the problem-solving process of mathematically gifted students in the context of problem-solving strategies (Öztelli Ünal, 2019, Koç Koca & Gürbüz, 2021).

Several recommendations can be made in light of the research findings. It is believed that structuring mathematical problems and tasks for gifted students in such a way that they can perform mathematical abstraction will aid in revealing the students' inherent talents. Additionally, self-regulation education for students can aid in their mathematics course success.

Individual interviews were conducted to ascertain the process by which structure is constructed and consolidated within the scope of the current study. Studies that observe gifted students in the classroom have the potential to shed light on the social dimension that contributes to the abstraction context. However, classroom studies may provide an opportunity to identify resource management strategies that are not covered in the current research. This circumstance is significant because it enables the definition of another variable that can affect the abstraction process. Additionally, in future studies, problem-solving sessions with a larger sample size can be used to compare students' abstraction processes.

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REFERENCES

Azevedo, R., Guthrie, J. T., & Seibert, D. (2004). The Role of Self-Regulated Learning in Fostering Students' Conceptual Understanding of Complex Systems with Hypermedia. *Journal of Educational Computing Research*, *30*(1-2), 87-111. https://doi.org/10.2190/DVWX-GM1T-6THQ-5WC7

Bikner-Ahsbahs, A. (2004). *Towards the emergence of constructing mathematical meanings*, Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2, 119-126.

Boekaerts, M., & Corno, L. (2005). Self-regulation in the classroom: A perspective on assessment and intervention. *Applied Psychology: An International Review, 54*(2), 199-231. https://doi.org/10.1111/j.1464-0597.2005.00205.x

Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F., (2009). *Bilimsel araştırma yöntemleri* [Scientific research methods]. Ankara: Pegem Publishing.

Dreyfus T., Hershkowitz R., & Schwarz B. (2015). *The nested epistemic actions model for abstraction in context: Theory as methodological tool and methodological tool as theory*. In: Bikner-Ahsbahs A., Knipping C., Presmeg N. (eds)

Approaches to Qualitative Research in Mathematics Education. Advances in Mathematics Education. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-9181-6_8

Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *Journal of Mathematical Behavior, 23,* 271-300. https://doi.org/10.1016/j.jmathb.2004.06.002

Efklides, A. (2019). Gifted students and self-regulated learning: The MASRL model and its implications for SRL. *High Ability Studies*, *30*, 79-102.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*(3), 163-176. https://doi.org/10.2307/748391

Greenes, C. (1981). Identifying the gifted student in mathematics. *The Arithmetic Teacher*, *28*(6), 14-17.

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education 32,* 195-222. https://doi.org/10.2307/749673

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago: The University of Chicago Press

Koç Koca, A., & Gürbüz, R. (2021). Problem solving strategies used by gifted secondary school students to solve math problems. *Bolu Abant İzzet Baysal University Journal of Faculty of Education, 21*(1), 348-359. https://dx.doi.org/10.17240/aibuefd.2021.21.60703-862916

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast, 8* (1&2), 167–188.

Leikin, R., Koichu, B., & Berman, A. (2009). *Mathematical giftedness as a quality of problem-solving acts*. In R. Leikin, A. Berman, & B. Koichu (Eds.), Creativity in mathematics and the education of gifted students (pp. 115-127). Dordrecht, the Netherlands: Sense Publishers.

Marcou, A. (2007). *Coding strategic behaviour in mathematical problem solving* In D. Küchemann (Ed.), Proceedings of the British Society for Research into Learning Mathematics, 27, 1, (pp. 54-59).

Monaghan, J., & Ozmantar, M. F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics, 62*(3), 233-258. https://doi.org/10.1007/s10649-006-8753-x

Montague, M., & Applegate, B. (1993). Middle school students' mathematical problem solving: An analysis of think-aloud protocols. *Learning Disability Quarterly*, *16*(1), 19-32. https://doi.org/10.2307/1511157

Niederer, K., Irwin, R. C., Irwin, K. C., & Reilly, I. L. (2003). Identification of Mathematically Gifted Children in New Zealand. *High Ability Studies*, 14 (1), 71-84.

Öztelli Ünal, D. (2019). *Non-routine problem solving processes of Turkish mathematically gifted students* (Unpublished master's thesis). Boğaziçi University, Istanbul.

Panadero, E. (2017). A review of self-regulated learning: Six models and four directions for research. *Frontiers in Psychology*, *8*(422). https://doi.org/10.3389/fpsyg.2017.00422

Panadero, E., Klug, J., & Järvelä, S. (2016). Third wave of measurement in the self-regulated learning field: When measurement and intervention come hand in hand. *Scandinavian Journal of Educational Research, 60*(6), 723–735. https://doi.org/10.1080/00313831.2015.1066436

Pintrich, P. R. (1999). The role of motivation in promoting and sustaining self-regulated learning. *International Journal of Educational Research*, *31*, 459-470. https://doi.org/10.1016/S0883-0355(99)00015-4

Pintrich, P., & Groot, E.D. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*, 33-40.

Pintrich, P. R., Smith, D. A. F., Garcia, T., & Mckeachie, W. J. (1993). Reliability and predictive validity of the motivated strategies for learning questionnaire (Mslq). *Educational and Psychological Measurement*, *53*(3), 801-813. https://doi.org/10.1177/0013164493053003024

Pitta-Pantazi, D., Christou, C., Kontoyianni, K., & Kattou, M. (2011). A model of mathematical giftedness: Integrating natural, creative, and mathematical abilities. *Canadian Journal of Science, Mathematics and Technology Education*, *11*, 39-54.

Rosenbloom, P.C. (1960). Teaching gifted children mathematics. In: E. Torrance (Ed.) *Talent and Education: present status and future directions: papers presented at the 1958 Institute on Gifted Children* (pp 351-370). Minneapolis: University of Minnesota.

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Roth, A., Ogrin, S., & Schmitz, B. (2016). Assessing self-regulated learning in higher education: a systematic literature review of self-report instruments. *Educational Assessment, Evaluation and Accountability, 28*, 225-250. https://doi.org/10.1007/s11092-015-9229-2

Schoenfeld, A. (2007). What is mathematical proficiency and how can it be assessed? In Schoenfeld, A. (Ed.) *Assessing mathematical proficiency* (pp. 59-74) Cambridge: Cambridge University Press.

Singer, F. M., Sheffield, L. J., Freiman, V., & Brandl, M. (2016). *Research on and activities for mathematically gifted students*. In research on and activities for mathematically gifted students (pp. 1-41). Springer International Publishing.

Sriraman, B. (2003). Mathematical giftedness, problem solving, and the ability to formulate generalizations: The problem-solving experiences of four gifted students. *Journal of Secondary Gifted Education*, *14*(3), 151-165. https://doi.org/10.4219/jsge-2003-425

Swanson, H. L. (1992). The relationship between metacognition and problem solving in gifted children. *Roeper Review: A Journal on Gifted Education*, *15*(1), 43 48. https://doi.org/10.1080/02783199209553457

Tsamir, P. & Dreyfus, T. (2005). How fragile is consolidated knowledge? Ben's comparisons of infinite sets. *Journal of Mathematical Behavior*, *24*, 15-38. https://doi.org/10.1016/j.jmathb.2004.12.001

Veenman, M. V. J. (2011). Alternative assessment of strategy use with self-report instruments: a discussion. *Metacognition Learning* *6***,** 205–211, https://doi.org/10.1007/s11409-011-9080-x

Winne, P. H., & Perry, N. E. (2000). *Measuring self-regulated learning.* In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (p. 531–566). Academic Press. https://doi.org/10.1016/B978-012109890-2/50045-7

Yin, R. K. (2003). *Case study research: Design and methods.* USA: Sage.

Zimmerman, B. J. (1990). Self-regulated learning and academic achievement: An overview. *Educational Psychologist*, *25*(1), 3-17. https://doi.org/10.1207/s15326985ep2501_2

Zimmerman, B. J., & Martinez-Pons, M. (1990). Student differences in self-regulated learning: Relating grade, sex, and giftedness to self-efficacy and strategy use. *Journal of Educational Psychology*, *82*(1), 51-59. https://doi.org/10.1037/0022-0663.82.1.51