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EVALUATION OF THE MATHEMATICAL THINKING PROCESSES OF A GIFTED AND 4TH GRADE STUDENT ON GEOMETRY

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Abstract

In this study, the mathematical thinking processes of a gifted 4th grade student regarding geometry were examined. Based on the thought that the geometric thinking level of the student will affect his mathematical thinking skills, it was determined that his geometric thinking level is the second level. While studying mathematical thinking processes, the theoretical framework of the three worlds of mathematics was used. According to this theoretical framework, mathematical thinking is divided into three levels: the conceptual embodied world, the perceptual symbolic world and the axiomatic formal world. While examining the mathematical thinking processes of this student named Alp, two semi-structured interviews were conducted. In the first, his prior knowledge of geometry was questioned and a verbal expression of triangle inequality was presented. In the second interview, an activity that is expected to create knowledge structures about triangle inequality was presented. As a result of the research, it was seen that Alp was able to carry out a thinking process towards proof processes, even at a simple level. His thinking skills are generally limited to the conceptual embodied world dimension.

Keywords: Gifted student, mathematical thinking, three worlds of mathematics

INTRODUCTION

Gifted students can be described as individuals with special academic skills, leadership qualities and creative thinking (Davis and Rimm, 2004; Gardner, 1993; Guilford, 1967; Kirk and Gallagher, 1989; Renzulli, 2003; Sternberg, 2003). Giftedness in mathematics can be defined as individual talent that manifests itself in mathematics (Krutetskii, 1976). Krutetskii (1976) describes these capabilities as the acquisition, processing and retention of mathematical information. Individuals with mathematically gifted abilities demonstrate their mathematical thinking skills with extraordinary speed and accuracy. In addition, these individuals can see the different relationships between the concepts (Heid, 1983). Individuals with mathematical giftedness focus on how and why a problem is solved rather than how it is solved (Sheffield, 1994).

Students' judgments about concepts in mathematics are related to mathematical thinking (Doğan, Güner, 2012). During mathematical thinking, mathematical processes such as reasoning, problem solving, prediction and abstraction are applied (Henderson, 2002). Mathematical thinking is individual. It is an individual process. It is the process of creating new concepts within the knowledge structures of individuals.

Geometric thinking is part of mathematical thinking. Geometric thinking is the process by which individuals perceive and interpret shapes. When the individual first encounters geometric concepts, he/she tries to make sense of it in his/her mind. He/she analyzes the properties of the shapes and divides the shapes into groups. In short, he/she creates structures in his mind for geometric concepts and relationships. This process is a geometric thinking process. In geometric thinking, there is a transition from the physical world to the abstract world (Akarsu Yakar, 2019). According to Gündoğdu Alaylı (2012), geometric relations and structuring mathematical relations are related. All levels involved in mathematical thinking processes are also valid for geometric thinking.

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It is seen in the literature that the development of geometric thinking is generally based on the Van Hiele approach (Alyeşil, 2005; Bobango, 1988; Fidan, 2009; Gündoğdu Alaylı, 2012; Olkun, Toluk, Durmuş, 2002; Özcan, 2012; Usiskin, 1982). Van Hiele (1986) states that the development of geometric thinking in children takes place in five stages. These; visual level, analytical level, informal deduction (inference based on experience), formal deduction (inference) and the most advanced level. According to this theory, thinking proceeds sequentially. According to the first level, individual can name the shapes and make measurements. At the second level, the individual analyzes and explains the properties of shapes. At the third level, he/she can compare shapes and classify shapes. At the fourth level, the individual can make abstract thinking and make a geometric proof with the help of axioms. At the fifth level, he/she can put forward his/her own theorems. According to the studies in the literature, secondary school students generally perform geometric thinking between the 1st and the 3rd level (Breen, 2000; Gündoğdu Alaylı, 2012; Fidan, 2009; Fuys, 1985; Karakarçayıldız, 2016; Özcan, 2012). Therefore, at the beginning of the study, it was thought that the geometric thinking level of the gifted 4th grade student would also be within these ranges.

In this study, while examining student's mathematical thinking skills, the theoretical framework of the three worlds of mathematics was discussed. The three worlds of mathematics deal with mathematical thinking in three stages. These phases are assumed to occur sequentially (Tall, 2007). Each world of thinking expresses the transition from concrete thinking to abstract thinking. According to this theory, the first thinking phase is the conceptual-embodied world. In this world of thinking, the individual expresses the concrete properties of objects. In this world of thinking, the visual-spatial properties of objects are at the forefront (Jukić, Brückler, 2014). The second thinking phase is the proceptual-symbolic world. In this world of thinking, there is a concept-thinking phase during the process. It includes expressing the concept with symbols and algebraic thinking. The individual expresses the concept symbolically as a result of repetitive actions. The third stage of thinking is the axiomatic-formal world. Proof is involved here. The individual expresses the concepts in their own sentences. He/she creates his/her own definitions of concepts as a result of mathematical proof (Akarsu Yakar, 2019).

When the researches are examined (Jukić, Brückler, 2014; Kashefi, Ismail, Yusof, 2010; Tall, Lima, Healy, 2014; Vandebrouck, 2011), there is no field study that eliminates the three worlds theory of mathematics and geometric thinking together. Therefore, the examination of the theoretical framework of the three worlds of mathematics within the geometry learning area has formed the importance of this research. In addition, there has been no research that addresses the theoretical framework of the three worlds of mathematics at primary school level. According to the theoretical framework of the three worlds of mathematics, elementary school students cannot be expected to fully realize the axiomatic formal world-scale thinking process. However, in the context of mathematical thinking, it is thought that a gifted 4th grade student can develop a simple proof process. The aim of the research is to reveal the thinking process that the student will perform towards the proof process. Therefore, considering that he can express the proof processes even at a simple level and can use his own definitions to express the concepts, this dimension is also discussed in the study.

METHODS

This study was designed with a mixed pattern model. In other words, quantitative and qualitative designs were used together. Quantitative design was used to determine the research participant according to his geometric thinking levels. Singular scanning method was determined as a quantitative research design. Qualitative design was used to determine student's mathematical thinking skills. The case study was chosen as a qualitative research design. Student's mathematical thinking skills were determined using the semi-structured interview technique.

Purposeful sampling method was used to determine the participant of the study. Students' geometric thinking level and giftedness were considered as criteria. The "Geometric

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Thinking Levels Scale" developed by Alyeşil (2005) was used to determine student's geometric thinking levels. The scale was prepared by using the Van Hiele Geometry Test. The scale consists of 20 questions in total, 5 questions for each level. The alpha-reliability coefficient of the scale is 0.81. It was accepted that the students showed that level of thinking as a result of correctly answering four of the five questions at each level.

This scale was applied to gifted and 4th grade student in Kocaeli province Izmit district. In this study, only a gifted 4th grade student with a level of geometric thinking at the second level was selected. The student was given the nickname Alp. Alp's gifted diagnosis was made in the field of general mental talent and he is studying in science and art center. He has a basic knowledge of algebra and the concept of variable.

Two semi-structured interviews was conducted to determine the mathematical thinking skills of the student. In the interview questions, it was aimed to reveal the mathematical thinking skills of the student towards geometry. In the first interview, the student was first asked to explain the triangle. Thus, it was aimed to reveal the knowledge structures of the student about the triangle. Then he was asked to explain why the sum of the dimensions of the interior angles of the triangle is 180^0. Finally, a verbal statement about the conceptual structure of triangle inequality, which is among the 8th grade achievements, was presented and asked to question its accuracy. In the second interview, an activity on triangle inequality prepared by Akarsu Yakar (2019) was presented. In this activity, the student was expected to form the triangle inequality in the process.

Two semi-structured interviews took about 45 minutes. In the analysis of the data, the theoretical framework of the three worlds of mathematics was discussed. Student responses are evaluated in three thinking worlds.

The audio recordings of the interviews made in this study were transferred to the computer by the researchers. In the process of ensuring the validity and reliability of the semi-structured interview process, the variation method (Creswell, 2013) was used. Interview, document review and observation process formed the variation method. The documents of the audio recordings of the interview process and student answer sheets were evaluated and analyzed together.

RESULTS

In this section, Alp's mathematical thinking skills were evaluated based on student responses in semi-structured interviews. The student was first asked to explain the triangle. The interview is as follows.

Researcher: *Can you tell me about the triangle?*

Alp: *Hmm ... the triangle has three sides. it also has three corners.*
Researcher: *Does every shape with three sides or three corners show a triangle?*
Alp: *It should be closed. There must also be three interior angles.*

The aim of the meeting was to reveal Alp's existing knowledge structures about the triangle. Alp said that a triangle should have three sides, three corners and three interior angles. In addition, he was able to express the feature of being closed, which is one of the geometric shape features.

Then Alp was asked why the sum of the dimensions of the interior angles of the triangle is 180 degrees. The interview process is as follows.

Researcher: *"The sum of the dimensions of the interior angles of the triangle is 180 degrees." Do you think his statement is correct?*

Alp: *Yes true.*
Researcher: *So why? Can you show it correct?*
Alp: *Let's call it a triangle. The interior angles are 50, 60, 70.The sum is 180.*

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Researcher: *So why 180, not 140? Why did you choose the dimensions of the interior angles that way?*
Alp: *I have never thought about that. Can i think?*
Researcher: *Of course.*
(4 minutes later)
Alp: *Now I'm going to draw a triangle. It will also be an equilateral triangle. Look now this is 4 cm. This is 4. Then this is 4. If I measure now. Yes, it's 60 degrees. 60 degrees here. Here too. Their total is 180. It turned out to be correct.*

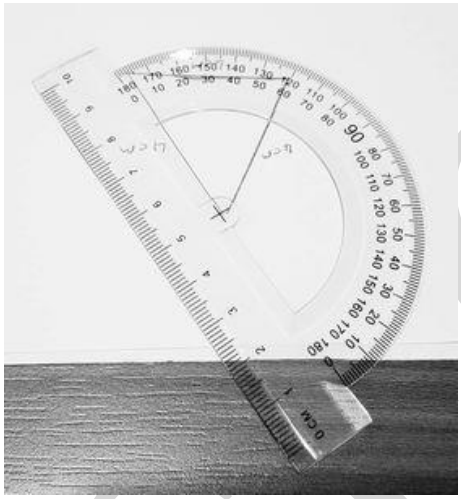

Figure 1. The figure drawn by Alp while expressing the sum of the dimensions of the interior angles of the triangle

Alp was able to explain why the sum of the interior angles of the triangle is 180 degrees by drawing an equilateral triangle. In the thinking process, he first realized the process of thinking about the conceptual embodied world dimension. He tended to explain the statement given before. So he wanted to describe the conceptual structure. Later he showed the skill of experimenting with numbers. All these skills can be thought of as skills belonging to the conceptual embodied world dimension. Then he developed a proof process to show the correctness of the conceptual structure. Therefore, he realized an axiomatic thinking process towards the formal world dimension.

Finally, in the interview with Alp, a verbal statement about the triangle inequality was presented to him. It is expected to show accuracy. The interview process is as follows.

Researcher: The length of any side in a triangle is less than the sum of the other two side lengths but greater than the difference. Provided that you subtract the short side length from the long edge length while getting the difference. Do you think his statement is correct?

Alp: *May be. I can actually draw a triangle and measure it.*

Researcher: *So do you want to think of a triangle with one side 9cm and the other 5cm long?*

Alp: *(He drew a triangle) Now this edge is 9, let this side be 5. (drawing) Yes let's measure it ok this is 10. yes it is true.*

Researcher: *Can you explain?*

Alp: *This is the 9 and this is the 5. He says it will be less than the sum of the two. Yes, little. because the two add up to 14. the differences are also 4. Bigger than 4. that's right.*
Researcher: *Is it always true?*
Alp: *So I drew it and it was correct. Is it always true? Yes it must be right. I don't know. I have to try all the time.*

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Researcher: *So how would you think of this expression if you didn't know the side lengths of the triangle?*

Alp: *If I didn't know the side lengths I guess I wouldn't*
Researcher: *If you gave letters to the lengths of the sides, for example if this was a, this was b, and this was c?*
Alp: *Then I couldn't.*

Alp was able to explain the verbal statement regarding the triangle inequality; however, it has only shown its accuracy with an example. He had difficulty developing a proof-of-accuracy process. The symbolic expression was also unable to write. It can be said that the thinking process is limited to the conceptual embodied world dimension.

In the second interview, an activity was presented to Alp. In this activity, he was expected to discover the triangle inequality. The meeting process for the event is as follows.

Researcher: *"One day, his teacher gives Efe a project assignment. According to the project homework, Efe has to design a triangular garden. Efe's teacher states that the length of one side of the garden should be 9 cm and the length of the other side should be 5 cm.*

What kind of path does Efe follow when he thinks about how to draw the triangle?
Alp: *Draws the lengths. I just drew it before.*
Researcher: *Do you think Efe can choose the third edge as 4 cm?*
Alp: *There are 4 differences between 5 and 9. It can happen. We need to look.*
(Trying again and again)
Doesn't seem to be less than 5. It just doesn't seem to coincide.

Researcher: *Do you think Efe can choose the third edge as 14 cm?*

Alp: *This time their total is 14. we can try again. But there was an expression saying it could not be in our previous meeting. Give me a try*
(trying)
Like it's not happening.

Researcher: *If we want to generalize this, how would you explain it?*

Alp: *I'm not sure exactly. but if we say it like this. It doesn't work if we subtract two of them, it doesn't work if we add them together. In fact, it seems to overlap anyway. If we add 5 to 9, it makes 14. the edges overlap. yes yes edges overlap. No way. yes they don't have sums. If we take it out, it will not.*

Alp made a process of thinking about creating the triangle inequality by testing the lengths of the sides of the triangle. He implemented the conceptualization process of triangle inequality by applying the instructions included in the activity. The instructions in the process of the activity helped him. He carried out the process of thinking towards the proof process but his thinking skills were limited to the conceptual embodied world dimension.

DISCUSSION and CONCLUSIONS

One of the biggest difficulties in mathematics education is the process of making sense of abstract concepts. In this process, we first use our concrete perceptions. We first think concretely. We evaluate abstract concepts within our existing mind schemas. In this study, the mathematical thinking skills of a gifted 4th grade student regarding some geometric structures were examined. When the relevant literature was examined, it was seen that the mathematical thinking processes of individuals were generally examined in terms of different theoretical frameworks such as APOS, SOLO taxonomy, RBC theory (Açıl, 2015; Bağdat, Saban, 2014; Köse, 2018; Mudrikah, 2016; Türnüklü, Özcan, 2014). In this study, the three worlds of mathematics are used as the theoretical framework.

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According to the theoretical framework of the three worlds of mathematics, mathematical thinking takes place in three stages. These stages follow a path from concrete to abstract. In this study, the reason why the gifted and geometric thinking level of the 4th grade student was chosen as the second level is to evaluate the progress of thinking processes towards the axiomatic formal world dimension. At primary school level, students are not expected to be able to demonstrate fully. However, it was thought that they could show the proof process within their thinking processes. Therefore, the results of the study also support this idea. The student was able to develop a proof process while explaining why the sum of the dimensions of the angles of the triangle is 180 degrees. Based on the example, he was able to show the accuracy of the statement and generalize the statement. He developed a thinking process from the conceptual embodied world dimension to the axiomatic formal world dimension.

Triangle inequality is among the 8th grade achievements according to the MEB (2019) mathematics curriculum. The student was able to interpret the verbal statement about inequality correctly. He wanted to explain the expression with an example and made a drawing. He showed the accuracy of the statement through an example. However, he could not generalize the geometric structure and show it symbolically. Alp has some prior knowledge about algebra. However, mathematical thinking skills did not reach the level of creating symbolic expressions. Therefore, the thinking process was limited to the conceptual embodied world dimension. Alp's thinking process could not reach the perceptual symbolic world and the axiomatic formal world dimension.

Within the activity that is expected to create the triangle inequality, there is a progress in mathematical thinking skills from the conceptual embodied world to the axiomatic formal world. Since he was a 4th grade student, his symbolic expression skills were not questioned. However, it has been observed that it reveals the ability to prove through trial and error. The student was able to create a knowledge structure about triangle inequality in the activity.

It was thought that the geometric thinking level and giftedness of the student affected his mathematical thinking skills. Geometric thinking is part of mathematical thinking. Their processes are similar. Both involve a thinking process from the concrete to the abstract. Individuals with second-order geometric thinking skills can explore the properties of geometric shapes and their parts. Having a second level geometric thinking level may have enabled the student to perceive the given instructions correctly. In addition, the giftedness of the student positively affected the thinking processes he developed towards geometry. According to Krutetskii (1976), giftedness in mathematics includes the components of acquiring, processing and remembering mathematical knowledge. These components are also thought to affect the student's geometric and mathematical thinking skills.

As a result of the research, it is thought that examining the mathematical thinking skills of the students is important in revealing their existing knowledge structures. Therefore, it is thought that it is important to enable them to explore the concepts in the activities instead of directly presenting them to the students in the teaching process. According to Fidan and Türnüklü (2010), instead of giving geometric concepts directly to the student, the student should be encouraged to find and create these concepts and should be given education appropriate to their level. Measurement and evaluation processes can also be evaluated in this context. Since it is thought that the theoretical framework of the three worlds of mathematics can be studied in the field of geometry, similar studies are recommended.

Thinking dimensions progress sequentially according to the three worlds of mathematics. As skills in mathematical thinking increase, mathematical thinking levels increase. Therefore, it is thought that the basic mathematical thinking skills of the student studying in the 4th grade of primary school also affect his mathematical thinking level. Monitoring the progress of mathematical thinking level in the future is thought to be important in terms of examining the development of mathematical thinking. In this context, studies involving processes can be conducted.

REFERENCES

Açıl, E. (2015). *Ortaokul 3. sınıf öğrencilerin denklem kavramına yönelik soyutlama süreçlerinin incelenmesi: Apos teorisi [Investigation of middle school 7rd graders abstraction processes of the concept of equation: APOS theory]* Unpublished Doctorate Thesis. Ataturk University, Institute of Education Sciences, Erzurum.

Akarsu Yakar, E. (2019). *Ortaokul öğrencilerinin matematiksel düşünme süreçlerinin ve matematiksel dil becerilerinin matematiğin üç dünyası kuramsal çerçevesi açısından incelenmesi [Examining the mathematical thinking processes and mathematical language skills of secondary school students in terms of the theoretical framework of the three worlds of mathematics]*. Unpublished Doctorate Thesis, Dokuz Eylul University, Institute of Educational Sciences, İzmir.

Alyeşil, D. (2005). *Kavram haritaları destekli ve problem çözme merkezli geometri öğretimi 7. sınıf öğrencilerinin geometrik düşünme düzeyleri üzerindeki etkileri [The effect of geometry learning with the method of conception charts supported problem solving to the 7th class students' geometry thinking level]* Unpublished Master's Thesis. Dokuz Eylul University, Institute of Education Sciences, İzmir.

Bağdat O., & Saban P. (2014). İlköğretim 8. sınıf öğrencilerinin cebirsel düşünme becerilerinin Solo taksonomisi ile incelenmesi [Investigation of the 8th grade students' algebraic thinking skills with solo taxonomy]. *International Journal of Social Science*, 26 (2), 473-496.

Bobango, J. C. (1988). Van Hiele levels of geometric thought and student achievement in standard content and proof writing: The effect of phase-based instruction. *Dissertation Abstract Index*, 48 (10).

Breen, J. J. (2000). Achievement of Van Hiele level two in geometry thinking by eight grade students through the use of geometry computer-based guided instruction. *Dissertation Abstract Index*, 60 (07).

Creswell, J. W. (2003). *Research design: Qualitative, quantitative and mixed methods approaches*. California: Sage Publications.

Davis, G. A., & Rimm, S. B. (2004). *Education of the gifted and talented*. Boston, MA: Pearson Education Press.

Doğan, M., & Güner, P. (2012). *İlköğretim matematik öğretmen adaylarının matematik dilini anlama ve kullanma becerilerinin incelenmesi [Examination of pre-service primary school mathematics teachers' skills of understanding and using mathematics language]*. Paper presented at the X. National Science and Mathematics Education Congress, Niğde University, Niğde.

Fidan, Y. (2009). *İlköğretim 5. sınıf öğrencilerinin geometrik düşünme düzeyleri ve buluş yoluyla geometri öğretiminin öğrencilerin geometrik düşünme düzeylerine etkisi [The geometric thinking levels of primary grade 5 students and the effect of geometry teaching with discovery learning on geometric thinking levels of students]*. Unpublished Doctorate Thesis. Dokuz Eylul University, Institute of Education Sciences, İzmir.

Fuys, D., (1985). Van Hiele levels of thinking in geometry. *Education and Urban Society*, 17(4), 447-462.

Gardner, H. (1993). *Frames of mind: The theory of multiple intelligences*. New York: Basic.

Guilford, J. P. (1967). *The nature of human intelligence*. New York: Mcgraw-Hill.

Gündoğdu Alaylı, F. (2012). *Geometride şekil oluşturma ve şekli parçalarına ayırma çalışmalarında ilköğretim 6. 7. ve 8. sınıf öğrencilerinin düşünme süreçlerinin incelenmesi ve bu süreçteki düzeylerinin belirlenmesi [The investigation of thinking process of primary 6th ,7th and 8th grade students in the studies of composing and decomposing shapes in geometry and determination of their levels in this process]*. Unpublished Doctorate Thesis. Dokuz Eylul University, Institute of Education Sciences, İzmir.

Heid, M. K. (1983). Characteristics and special needs of the gifted student in mathematics. *Mathematics Teacher*, 76, 221–226.

Henderson, P. (2002). Materials development in support of mathematical thinking. *ACM SIGCSE Bulletin*, 35(2), 185-190. Erişim adresi: http://Blue.Butler.Edu/Phenders/İticse2002wg.Rtf

Jukić, L., & Brückler, F.M. (2014). What Do Croatian Pre-Service Teachers Remember from Their Calculus Course?. *Issues in the undergraduate mathematics preparation of school teachers*, 1,1-15.

Karakarçayıldız, R. Ü. (2016). *7. Sınıf öğrencilerinin geometrik düşünme düzeyleri ile çokgenleri sınıflama becerileri ve aralarındaki ilişki [Geometrical thinking levels and polygons classification skills of 7 th grade students and relationship between them]*. Unpublished Master's Thesis. Eskisehir Osmangazi University, Institute of Education Sciences, Eskişehir.

Kashefi, H., Ismail, Z., & Yusof, Y. M. (2010). Obstacles in the learning of two-variable functions through mathematical thinking approach. *Pocedia Social and Behavioral Sciences*, 173-180.

Kirk, S. A. & Gallagher, J. J. (1989). *Educating exceptional children*. Boston, MA: Houghton Mifflin

Köse, O. (2018). *Üst düzey uzamsal yeteneğe sahip matematik öğretmen adaylarının düşünme yapılarına göre SOLO taksonomisi düzeylerinin belirlenmesi [Determination of SOLO taxonomy levels of mathematics teacher candidates with high level spatial ability according to thinking structures]* Unpublished Master's Thesis. Selcuk University, Institute of Education Sciences, Konya.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago, IL: University of Chicago Press

Milli Eğitim Bakanlığı (2019). *Matematik dersi öğretim programı [Mathematics lesson curriculum]*. Ankara: Talim ve Terbiye Kurulu.

Mudrikah, A. (2016). Problem-based learning associated by action-process-object-schema (APOS) theory to enhance students' high order mathematical thinking ability. *International Journal of Research in Education and Science (IJRES)*, 2(1), 125- 135.

Olkun, S., Toluk, Z., & Durmuş, S. (2002, Eylül). *Sınıf öğretmenliği ve matematik öğretmenliği öğrencilerinin geometrik düşünme düzeyleri [Geometric thinking levels of primary school and mathematics teaching students]*. Paper presented at the 5th National Science and Mathematics Education Congress, ODTÜ, Ankara.

Özcan, B. N. (2012). *İlköğretim öğrencilerinin geometrik düşünme düzeylerinin geliştirilmesinde bilgiyi oluşturma süreçlerinin incelenmesi [The investigation of knowledge construction process of primary school students while developing geometric thinking levels]*. Unpublished Doctorate Thesis. Dokuz Eylul University, Institute of Education Sciences, İzmir.

Renzulli, J. S. (2003). Conception of giftedness and its relationship to the development of social capital. N. Colangelo ve G. A. Davis (Editörler.) *Handbook of gifted education* (75–87). Boston MA: Allyn and Bacon, Inc

Sheffield, L. J. (1994). *The development of gifted and talented mathematics students and the National Council of Teachers of Mathematics Standards* (No. 9404). DIANE Publishing.

Sternberg, R. J. (2003). Giftedness according to theory of successful intelligence. N. Colangelo ve G. A. Davis (Editörler) *Handbook of gifted education* (88-99). Boston MA: Allyn and Bacon, Inc.

Tall, D. (2007). Developing a theory of mathematical growth. *Zdm Mathematics Education*, 39, 145-154.

Tall, D., Lima, R. N., & Healy, L. (2014). Evolving a three-world framework for solving algebraic equations in the light of what a student has met before. *The Journal of Mathematical Behavior*, 34, 1-13.

Türnüklü, E., & Özcan, B. (2014). Öğrencilerin geometride RBC teorisine göre bilgiyi oluşturma süreçleri ile Van Hiele geometrik düşünme düzeyleri arasındaki ilişki: Örnek olay çalışması [*The relationship between students' construction of geometric knowledge process based on rbc theory and van hiele geometric thinking levels: case study]*. *Mustafa Kemal University Journal of Graduate School of Social Sciences*, 11 (27), 295-316.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry, Final Report, Cognitive Development And Achievement in Secondary School Geometry Project*. Chicago: University Of Chicago.

Vandebrouck, F. (2010). *Students conceptions of functions at the transition between secondary school and university*. Proceedings of the Conference CERME 7, University of Rzeszow, Poland.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Pr.